Exercise 1

(a) Calculate the carrier frequency corresponding to the following optical wavelengths:

$\lambda = 633$ nm, the wavelength of the Helium Neon (HeNe) laser

$\lambda =$800 nm, the wavelength of the 1st generation of transmission systems

$\lambda =$1300 nm, the wavelength of the 2nd generation of transmission systems

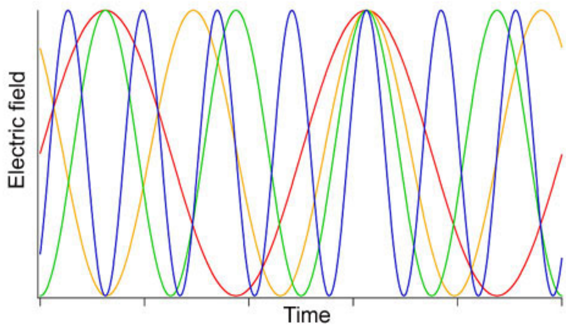
$\lambda = 1550$ nm, the wavelength of the modern transmission systems

(b) What is the photon energy in each case? Calculate the energy in electron volt (eV) given that $1\ \mathrm{J} = 6.24 \cdot 10^{18}\ \mathrm{eV}$.

(c) Given that a communication system can be operated at a bit rate up to 1% of the carrier frequency find the number of audio channels at 64 kb/s that could be transmitter over a microwave carrier at $\nu_{\mu wave} =$ 5 GHz or over an optical link at a wavelength of 1550 nm.

Exercise 2

One technique to make short pulses of light directly out of a cavity is called 'modelocking'. The different longitudinal modes of the cavity have a fixed phase relationship between each other so that the electric fields add constructively in one position of the cavity and destructively everywhere else (see the cartoon illustration below, with an exaggerated scale – the different modes have different oscillation frequencies shown by the colors).



A titanium-sapphire laser has a gain bandwidth of approximately 100 THz centered at 800 nm.

(a) When $\Delta\nu \ll \nu_0$ you can use the linear relationship between $\Delta\nu$ and $\Delta\lambda$ using the differentiation approach. Write this equation$

(b) What is the minimum possible pulse duration assuming transform-limited pulses and a Gaussian pulse shape. Use the linear approximation.

(c) If the pulse is instead sech²-shaped, how does the answer change? Use the linear approximation.

(d) Derive the exact expression for $\Delta\nu$ as a function of $\Delta\lambda$, and the central wavelength λ_0

(e) Plot (using matlab), $\Delta\nu$ vs $\Delta\lambda$ for $\Delta\nu$ between 1 GHz and 10^6 GHz, and for $\lambda_0 = 0.5\ \mu\mathrm{m}$ $1\ \mu\mathrm{m}$ and $1.5\ \mu\mathrm{m}$ (use a log-log to see better)

(f) From the graph, show that a Gaussian 2 fs pulse at $1\ \mu\mathrm{m}$ requires a bandwidth of approximately 850 nm !

Exercise 3

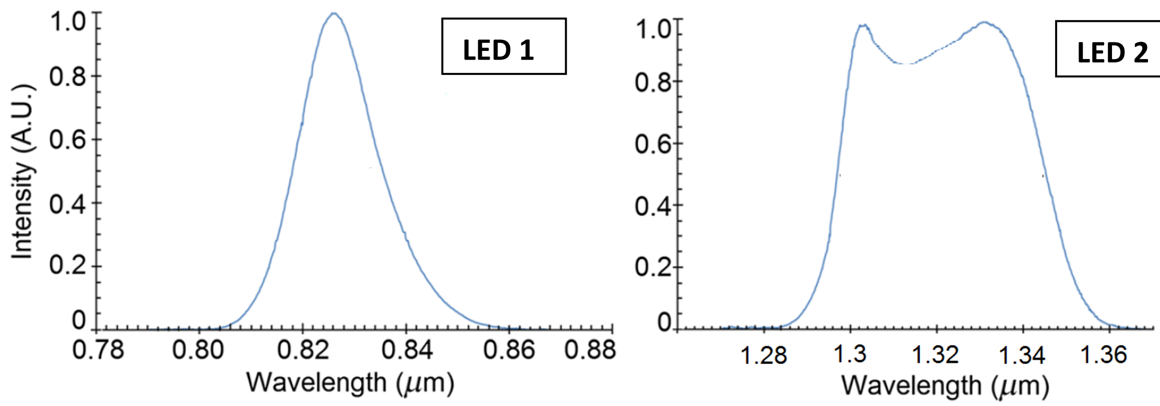
A 1550 nm digital communication system operates at a rate of 1 Gb/s. It receives an average of - 40 dBm at the output of the link. It is assumed that '1' and '0' bits are equally likely to occur (that is there is 50% of the time a bit '1' is received and the other 50% a bit '0' is received). You can also assume that a bit '0' contains no power.

(a) What is the power of a '1' bit in dBm and in mW?

(b) How many photons are received within each '1' bit ?

Exercise 4

LEDs are used for optical coherence tomography where the theoretical axial (depth) resolution is linked to the coherence length of the source: the shorter the coherence length the better the resolution. The spectra of two LEDs are shown below. Which LED would you use to have the best resolution ? (NOTE: this is a very simplified example, and many other parameters come into play in real systems.) (use linear relation between $\Delta\nu$ and $\Delta\lambda$).



Exercise 5

A ruby laser makes use of a 10 cm long ruby rod (n = 1.76), has a transition cross section is $\sigma(\nu_0) = 1.26 \cdot 10^{-20}\ \mathrm{cm}^2$, and operates on this transition at 694.3 nm. Both ends of the ruby rod are polished and coated so that each has a reflectance of 80%. Assume that there are no scattering or other extraneous losses.

As this laser is pumped, the medium goes from absorbing to amplifying. What is the required threshold population difference required to reach laser oscillation?

Exercise 6

In class, we derived that when lasing, the population difference inside a laser is clamped to: $N_{th} = \frac{1}{a\tau_p}$, with a a quantity associated with the probability that a carrier will capture a photon and give rise to stimulated emission. We want to find an expression for the quantity a.

(a) When a laser starts lasing, what happens to the gain?

(b) Using the answer from part (a), show that the quantity a is given by $a = \sigma(\nu_0)c$: where $\sigma(\nu_0)$ is the transition cross section and c the speed of light in the laser medium.

Exercise 7

A mode-locked laser produces pulses with an average power of 1 W, a repetition rate of 100 MHz, and a pulse duration of 50 fs.

(a) What is the energy per pulse?

(b) What is the peak power of the pulse?

Graded Exercise

Einstein coefficients – In class we have derived the probability densities of spontaneous and stimulated transitions under the assumption of monochromatic light having a photon flux density ϕ, such that the *probability density of an induced transition* was $W_i = \phi\sigma(\nu)$.

We will here carry similar analysis but with broadband light as done by Einstein in 1917, where he assumed energy exchange between atoms and radiation at thermal equilibrium through broadband radiation of spectral energy density $\varrho(\nu)$ (unit of $\mathrm{Jm^{-3}s}$) such as coming from a black body. Considering two levels 1 and 2, with 2 being an upper level, he named:

- the probability density of transition due to *spontaneous emission* A_{21}. Unit of A_{21} is s^{-1}.
- the *probability density of stimulated emissions* B_{12} and B_{21}, depending on the direction of the transition. Unit of B_{12} and B_{21} is $\mathrm{J^{-1}m^3s^{-2}}$

a. Considering the three possible atom-radiation interactions, write the rate equation for the population of the upper level N_2 $(i.e. \frac{dN_2}{dt})$ using Einstein *A* and *B* coefficients and assuming spectral energy density $\varrho(\nu)$. (Start by expressing the rate of change of N_2 for the three processes)

b. Given that the blackbody spectrum of radiation is given by $\varrho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{k_B T}\right)-1}$, and using Boltzmann distribution, show that at steady state $B_{12} = B_{21} \equiv B$ and that $\frac{A_{21}}{B} = \frac{8\pi h\nu^3}{c^3}$.

c. Based on the answer found in b. show that the rate of change of N_2 due to spontaneous emission always dominates the rate of change of N_2 due to stimulated emission at room temperature (*T* = 300 K and $k_B T$ = 25 meV) in the near infrared region where $h\nu \approx 1\ eV$. This is another way of understanding why lasing cannot occur under such condition and why pumping is required.

d. At what wavelength are the rates of spontaneous and stimulated emission equal in a two-level system at room temperature (*T* = 300 K)?